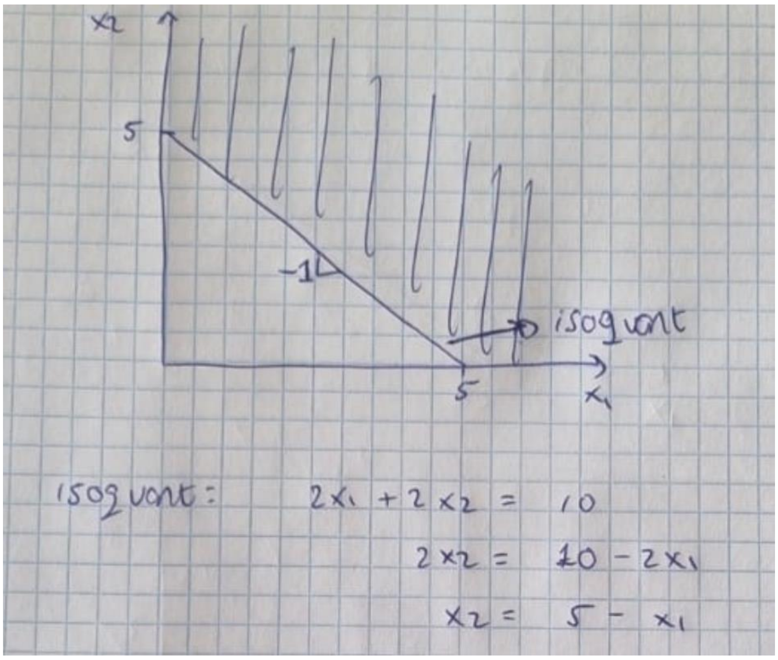
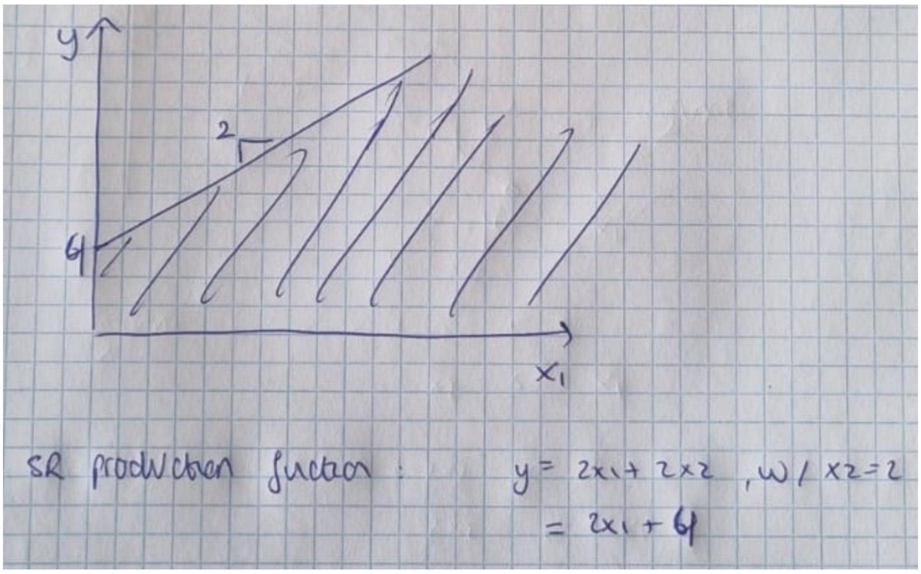
**1.1. (1 point)**

$$TRS = -\frac{\frac{df}{dx_1}}{\frac{df}{dx_2}} = -\frac{\beta}{\gamma}$$

**1.2. (1 point)**




**1.3. (1 point)**



**2.1.(1 point)**

The price $p$ enters the profit function directly and indirectly via $x(p,w)$, so taking the derivative towards the price yields:

$$\frac{\partial \pi\big(p,w,x(p,w)\big)}{\partial p} = \frac{\partial \pi(p,w,x)}{\partial p} + \frac{\partial \pi(p,w,x)}{\partial x}\frac{\partial x(p,w)}{\partial p}$$

Hence, when the price changes, there are two ways the profits are affected:

(1) Directly: when the price changes, profits change while $x$ remains constant.
(2) Indirectly: when the price changes, the firm changes its choice for $x$ ($\frac{\partial x(p,w)}{\partial p}$) and this change subsequently affects profits ($\frac{\partial \pi(p,w,x)}{\partial x}$).

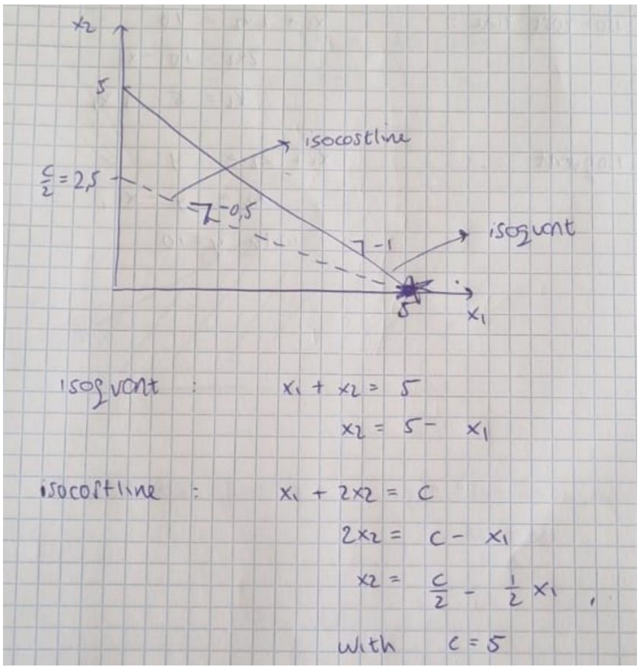
Applying the envelope theorem, we know that changes of an exogenous variable only affect the optimized function directly, even if the exogenous variable also enters the optimized function indirectly via endogenous choice variables.

Here, $p$ is the exogenous variable and $x = x(p,w)$ is the endogenous choice variable. Note that $x(p,w)$ is obtained via the first order condition:

$$\frac{\partial \pi(x,p,w)}{\partial x} = 0$$

Hence, when $x$ changes the profits do not change, since $x$ is already chosen optimally. This implies that the indirect effect (2) is zero, and only the direct effect (1) remains.

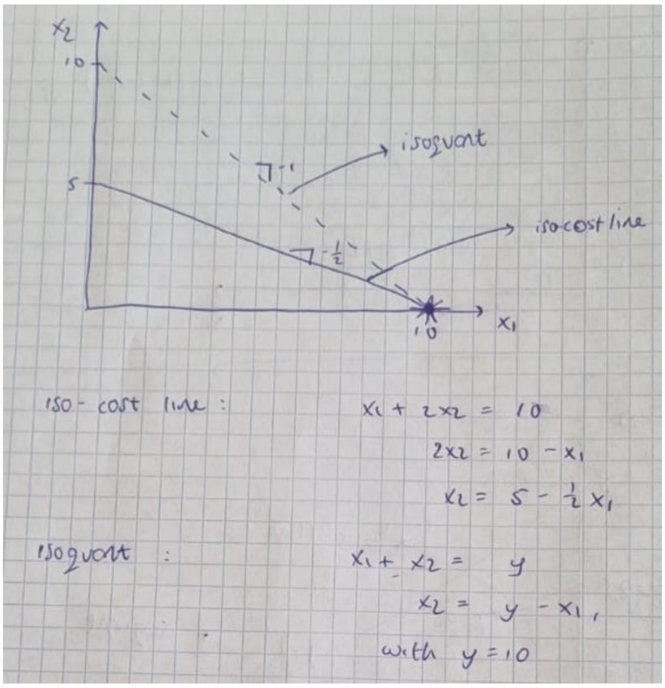
**2.2.(1 point)**




The conditional factor demands are $x_1 = 5$ and $x_2 = 0$.

The minimum costs are 5. That is, with $c = 5$ we have the lowest intercept of the iso-cost line that still gives us a combination of $x_1$ and $x_2$ that allows us to produce 5 units of output.

**2.3. (1 point)**




The conditional factor demands are $x_1 = 10$ and $x_2 = 0$.

Note that in contrast to question 2.2, here you need to fix the iso-cost line and get the highest intercept of the isoquant that can be achieved while costs are equal to 10.

**2.4. (1 point)**

In a consumer choice problem, the utility function would replace the production function, so $u = 1x_1 + 1x_2$, and the budget constraint would replace the cost function, so $1x_1 + 2x_2 = m$.

The answers to 2.2 and 2.3 imply that demand for good $x_2$ does not depend on income $m$: we have that $x_2 = 0$ for an income equal to 5 and 10. Hence, the income effect for good $x_2$ is zero.

**3.1. (1 point)**

Convexity of consumer preferences ensures that the Hicksian demand function is downwards sloping. When the price of good 1 increases, the budget line gets steeper (with $x_1$ on the horizontal axis), and so it touches the same indifference curve (with a convex upper contour set) at a lower $x_1$. Hence, when the price of good 1 increases, the demand for $x_1$ decreases while keeping utility constant.

**3.2. (1 point)**

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 to reach:

$$x_1(p_1, m) = \frac{m}{p_1}\alpha$$

$$x_2(p_2, m) = \frac{m}{p_2}(1-\alpha)$$

**3.3. (1 point)**

$$\frac{\partial x_1(p_1, m)}{\partial p_1} = -\frac{m}{p_1^2}\alpha$$

$$\frac{\partial x_2(p_2, m)}{\partial p_2} = -\frac{m}{p_2^2}(1-\alpha)$$

Both goods are ordinary if both derivatives are smaller than zero. Both derivatives are smaller than zero if:

$$\alpha > 0$$

$$(1-\alpha) > 0$$

Which implies

$$0 < \alpha < 1$$

**3.4. (1 point)**

1. fill the Marshallian demand functions into the utility function

2. simplify

$$v(\boldsymbol{p}, m) = \left(\frac{m}{p_1}\alpha\right)^{\alpha}\left(\frac{m}{p_2}(1-\alpha)\right)^{1-\alpha} = m\alpha^{\alpha}(1-\alpha)^{1-\alpha}p_1^{-\alpha}p_2^{\alpha-1}$$

**3.5. (1 point)**

To obtain lambda one needs to take the derivative of the indirect utility function towards $m$:

$$\frac{\partial v(\boldsymbol{p}, m)}{\partial m} = \lambda(\boldsymbol{p}) = \alpha^{\alpha}(1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{\alpha-1}$$

Plug in for $\alpha = \frac{1}{2}$, $p_1 = 1$, and $p_2 = 4$:

$$\lambda(\boldsymbol{p}) = 0.5^{0.5}(1-0.5)^{1-0.5} 1^{-0.5} 4^{0.5-1} = 0.25$$

This implies that if a consumer's income increases by 1 (so we relax the constraint by 1), then the consumer can obtain 0.25 additional util.

**4.1. (1 point)**

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 to reach:

$$x_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2$$

1. Write down the Lagrangian for the EMP

2. Take FOCs

3. Solve these FOCs for x1 (denoted by h1) to reach:

$$h_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2$$

**4.2. (1 point)**

With quasilinear utility we have that $x_1 = h_1$. The change in consumer surplus is the area to the left of the Marshallian demand curve $x_1$. The compensating and equivalent variation are the areas to the left of the Hicksian demand curve $h_1$. Since the Hicksian and Marshalian demand coincide, the compensating and equivalent variation and change in consumer surplus are the same. The equivalent and compensating variation are exact measures of welfare, whereas in general the change in consumer surplus is not. However, since these are equal with quasilinear utility, the change in consumer surplus can be used as an exact measure of welfare.

**4.3. (2 points)**

Here we calculate the change in consumer surplus. Start with the Marshallian demand function while assuming that $p_2 = 4$ as stated in the exercise, then:

$$x_1(p_1) = \frac{1}{4}\left(\frac{4}{p_1}\right)^2 = 4p_1^{-2}$$

Then we integrate the demand function for a price change from 1 to 2:

$$\int_1^2 x_1(p_1)\, dp_1 = \int_1^2 4p_1^{-2}\, dp_1 = -4p_1^{-1}|_1^2 = -2 - -4 = 2$$

Hence, the change in consumer surplus is 2.

**5.1. (2 points)**

There are two conditions for the long run equilibrium:

$Y(p) = X(p)$

$\pi_i = 0 \quad \forall\, i$

1. We derive the firms' supply function $y_i(p)$ for each firm i.

$mc_i(y) = \frac{dc_i(y)}{dy} = y,$

and since supply curve is $mc_i(y) = p$,

we have that $y_i(p) = p$.

2. We derive market supply, which is the sum over all firms m.

$Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} p = mp.$

3. We use the first condition to find equilibrium price and firm supply in terms of number of firms m.

$$mp = 40 - 2p$$

$$p = \frac{40}{m+2}$$

$$y_i(p) = p = \frac{40}{m+2}$$

4. We use the second condition to find the number of firms m so that profits are zero.

$$\pi_i = py_i(p) - c_i(y) = 0$$

$$\pi_i = \left(\frac{40}{m+2}\right)^2 - 0.5\left(\frac{40}{m+2}\right)^2 - 2 = 0$$

$$0.5\left(\frac{40}{m+2}\right)^2 = 2$$

$$\left(\frac{40}{m+2}\right)^2 = 4$$

$$\frac{40}{m+2} = 2$$

$$m = 18$$

Hence, in the long run there will be 18 active firms in this perfect competitive market.

**6.1. (1 point)**

The monopolist maximizes profits by choosing $Q$ such that $MR(Q) = MC(Q)$.

First, from the demand curve we find the inverse demand curve:

$$p(Q) = 50 - 0.5Q$$

Then we can write total revenue as:

$$TR(Q) = p(Q) \times Q = (50 - 0.5Q) \times Q = 50Q - 0.5Q^2$$

Then we find MR

$$MR(Q) = \frac{\partial TR(Q)}{\partial Q} = 50 - Q$$

And MC is

$$MC(Q) = \frac{\partial c(Q)}{\partial Q} = 5 + 2Q$$

So, profits are maximized when:

$$MR(Q) = MC(Q)$$

$$50 - Q = 5 + 2Q$$

$$3Q = 45$$

$$Q^* = 15$$

$$p^* = 50 - 0.5 \times 15 = 42.5$$

And profits are equal to:

$$\pi^* = p^* \times Q^* - c(Q^*)$$

$$\pi^* = 42.5 \times 15 - (50 + 5 \times 15 + 15^2) = 287.5$$

**6.2. (1 point)**

The monopolist would set the price equal to the willingness to pay for each consumer. Hence, the price would just trace out the (inverse) demand curve.

All the surplus of the market goes to the monopolist, and the consumer surplus will now be zero since the price is equal to the willingness to pay for each consumer. Hence, the monopolist benefits from perfect price-discrimination, whereas consumers do not.

An example of (perfect) price discrimination is uber/bolt taxi services. Here the price varies per consumer, depending on for instance the time of day a consumer requests the taxi service.